2.4–2.5 Test C Name________________________________

Use the rational root theorem to find all possible rational zeros. Then use synthetic division and algebra to find the remaing zeros.

1) $f(x) = 4x^3 - 28x^2 - x + 7$

From graph $x = 7$ is a zero

$p = \pm 1, \pm 7$

$q = \pm 1, \pm 2, \pm 4$

All possible Zeros

$\pm 1 \; \pm \frac{1}{2} \; \pm \frac{1}{4}$

$\pm 7 \; \pm \frac{7}{2} \; \pm \frac{7}{4}$

$$\begin{array}{r|rrrr} 7 & 4 & -28 & -1 & 7 \\ & & 28 & 0 & -7 \\ \hline & 4 & 0 & -1 & \boxed{0} \end{array}$$

$4x^2 - 1 = 0$

$4x^2 = 1$

$x^2 = \frac{1}{4}$

$x = \pm \frac{1}{2}$

Zeros

$x = 7$

$x = \frac{1}{2}$

$x = -\frac{1}{2}$

Find the requested function.

2) Find the factored form and standard form of the polynomial function with degree 3; and 3, $\frac{1}{6}$, and $\frac{7}{6}$ as zeros.

$y = (x-3)(6x-1)(6x-7)$ → Factored Form

$(x-3)(36x^2 - 48x + 7)$

$36x^3 - 48x^2 + 7x$

$-108x^2 + 144x - 21$

$y = 36x^3 - 156x^2 + 151x - 21$ → Standard Form

3) a. Find the factored form and standard form of the cubic function with the given table of values.

x	-7	-4	0	5
f(x)	0	0	-280	0

$(-7, 0)$ $(-4, 0)$ $(5, 0)$ $(0, -280)$

b. Then sketch a graph of the function using the information given and the end behavior.

$y = a(x+7)(x+4)(x-5)$

Let $x = 0$ $y = -280$ solve for a

$-280 = a(7)(4)(-5)$

$-280 = -140a$

$a = 2$

$y = 2(x+7)(x+4)(x-5)$ → Factored

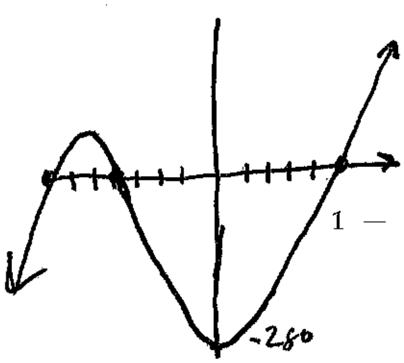
$= 2(x+7)(x^2 - x - 20)$

$x^3 - x^2 - 20x$

$= 2(x^3 + 6x^2 + 27x - 140)$

$7x^2 - 7x - 140$

$= 2x^3 + 12x^2 + 54x - 280$



Given the zeros and their multiplicities of a function, write the function in factored form
Find the y-intercept and end behavior of the function. Then sketch the graph.

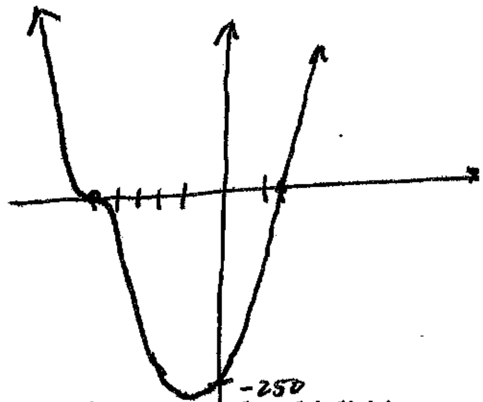
4) x = -5 (multiplicity of 3) x = 2 (multiplicity of 1)

$y = (x+5)^3(x-2)$

y-int $(0, -250)$

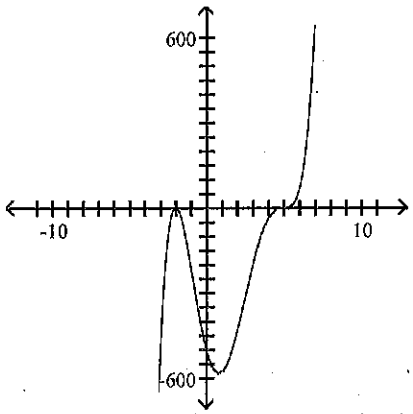
$x \to \infty, y \to \infty$

$x \to -\infty, y \to \infty$



Match the polynomial function graph to the appropriate zeros and multiplicities.

5)



A) –2 (multiplicity 3), 5 (multiplicity 2)
B) –2 (multiplicity 2), 5 (multiplicity 3)
C) –2 (multiplicity 3), 5 (multiplicity 3)
D) –2 (multiplicity 2), 5 (multiplicity 2)

Use the rational root theorem to find all possible rational zeros. Then use synthetic division and algebra to find the remaing zeros.

6) $f(x) = x^3 - 8x^2 + 4x + 48$

-2	1	-8	4	48
		-2	20	-48
	1	-10	24	$\boxed{0}$

$x^2 - 10x + 24$

$(x-4)(x-6)$

zeros $x = -2$

$x = 4$

$x = 6$

Write a polynomial function of minimum degree with real coefficients whose zeros include those listed. Write the polynomial in standard form.

7) 5i and $\sqrt{2}$

$x = 5i, -5i, \sqrt{2}, -\sqrt{2}$

$(x-5i)(x+5i)(x-\sqrt{2})(x+\sqrt{2})$

$(x^2+25)(x^2-2)$

x^4+23x^2-50

8) 4 and 2 - i

$x=4 \quad x=2-i \quad x=2+i$

$(x-4)(x-(2-i))(x-(2+i))$

$(x-4)(x-2+i)(x-2-i)$

$(x+4)(x^2-4x+5)$

$$\begin{array}{r} x^3-4x^2+5x \\ -4x^2+16x-20 \\ \hline x^3-8x^2+21x-20 \end{array}$$

Find all of the real zeros of the function. Give exact values whenever possible. Identify each zero as rational or irrational. Use the rational root theorem to find all possible rational zeros. Then use synthetic division and algebra to find the remaing zeros.

9) $f(x) = x^3 - 4x^2 - 7x + 28$

$$\begin{array}{r|rrrr} 4 & 1 & -4 & -7 & 28 \\ & & 4 & 0 & -28 \\ \hline & 1 & 0 & -7 & \boxed{0} \end{array}$$

$x^2-7=0$

$x^2=7$

$x=\pm\sqrt{7}$

$x=4$ Rational

$x=\pm\sqrt{7}$ Irrational

Write the function as a product of linear and irreducible quadratic factors, all with real coefficients.

10) $f(x) = x^4 - 2x^3 - 34x^2 - 2x - 35$

$$\begin{array}{r|rrrrr} -5 & 1 & -2 & -34 & -2 & -35 \\ & & -5 & 35 & -5 & 35 \\ \hline 7 & 1 & -7 & 1 & -7 & \boxed{0} \\ & & 7 & 0 & 7 & \\ \hline & 1 & 0 & 1 & \boxed{0} & \end{array}$$

x^2+1

$y = (x+5)(x-7)(x^2+1)$

Find all of the real zeros of the function given one of the irrational zeros. Give exact values whenever possible. Identify each zero as rational or irrational.

11) $f(x) = x^4 + 8x^3 + 4x^2 - 88x - 165$ $\quad x = \sqrt{11}$ is an irrational zero

$$\begin{array}{r|rrrrr} \sqrt{11} & 1 & 8 & 4 & -88 & -165 \\ & & \sqrt{11} & 11+8\sqrt{11} & 88+15\sqrt{11} & 165 \\ \hline -\sqrt{11} & 1 & 8+\sqrt{11} & 15+8\sqrt{11} & 15\sqrt{11} & \boxed{0} \\ & & -\sqrt{11} & -8\sqrt{11} & -15\sqrt{11} & \\ \hline & 1 & 8 & 15 & \boxed{0} & \end{array}$$

$\sqrt{11}(8+\sqrt{11})$

$8\sqrt{11}+11$

$\sqrt{11}(15+8\sqrt{11})$

$15\sqrt{11}+88$

$\sqrt{11}(15\sqrt{11}) = 165$

$x^2+8x+15=0$

$(x+5)(x+3)=0$

$x=-5 \quad x=-3$

Zeros

$x=-5$ Rational

$x=-3$ Rational

$x=\pm\sqrt{11}$ Irrational